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Incorporation of Memory Length Parameter in a Day To-Day Modelling Framework and Its Comparison With Stochastic User Equilibrium Model

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Abstract
Route choice modeling is a central component of transit assignment methods, playing a pivotal role in assessing how public transit travelers make decisions in the absence of external information. In reality, even identical travel experiences are perceived differently by different individuals due to variations in learning rate, risk-taking behavior, memory length, and other personal traits. This paper presents a simulation-based microscopic day-to-day (DTD) modeling framework that incorporates the memory length parameter alongside other stochastic elements in the route decision-making process. A congestion-sensitive cost function based on the Bureau of Public Roads (BPR) formulation is integrated into a logit-based route choice model. The day-to-day framework is developed for a dummy transit network and benchmarked against a conventional Stochastic User Equilibrium (SUE) model solved using the Method of Successive Averages (MSA). Sensitivity analyses are carried out with varying demand levels (100, 200, 240 passengers) and dispersion parameters (θ = 0.01, 0.05, 0.1, 0.5). Additionally, memory span tests are conducted in both laboratory and real-world field environments, and a regression model is developed to relate quantifiable personal traits (gender, age, income, education, frequency of transit use) to an individual's memory span. Key findings show that memory span is consistently lower in real-world settings than in controlled environments, that flow distributions in the DTD model are stochastic yet their averages converge to SUE flows, and that higher memory spans yield more stable route flows over time.
Keywords: *Route choice modeling, memory length, BPR function, day-to-day model, stochastic user equilibrium, transit assignment, logit model, perception deviation.*

1. Introduction

Transit assignment refers to the process by which a given origin–destination (O-D) passenger demand is distributed across the available transit routes of a network. Modeling the route choice of transit users constitutes one of the most important—and challenging—areas in transit assignment research. The unpredictability of human behavior, the limited knowledge travelers possess about network conditions, the uncertainty surrounding their perceptions of route attributes, and the variability in individual preferences all contribute to making route choice modeling a formidable task. Behavioral complexities include habits, learning process rates, risk attitudes, comfort criteria, and memory capacity.

Travel time is widely regarded as the most dominant criterion in route attractiveness. Most travelers prefer the route with minimum travel time; however, exact travel time cannot be known in advance because flows fluctuate continuously. Horowitz (1984) established that route choice is an outcome of previous subjective experiences. Passengers continuously undergo learning and adaptation during repeated trip-making: after traveling a route several times, a passenger begins to forecast travel time before departure. Most early approaches assumed that travelers possess perfect knowledge of network impedances—an unrealistic assumption.

The concept of memory span is critical in this context. Memory span refers to the maximum number of items a person can accurately recall. In route choice, this translates to the number of past travel time experiences a traveler can retain and use for informing future route decisions. If a passenger's memory span is five, they can analyze and utilize the five most recent travel-time experiences for route selection. This concept has been largely unexplored in the calibration of transit user memory length, and its explicit incorporation into a day-to-day stochastic framework represents the central contribution of this paper.

1.1 Objectives

- To study the factors influencing perceptional deviation of a person when no external information is provided.
- To incorporate the memory length parameter in a day-to-day model and implement it on a dummy network.
- To develop a Stochastic User Equilibrium (SUE) model for the same dummy network and compare it with the developed simulation model.
- To identify personal traits that influence the memory length of a person in a laboratory environment and develop a regression model for the same.
- To compare memory span measured in a controlled laboratory environment with the span effectively utilized in the field.

1.2 Scope

A frequency-based day-to-day stochastic model is developed in MATLAB, assuming exponentially distributed bus arrivals. The model is implemented on the De Cea and Fernandez (1993) dummy network and compared against the SUE model. Field and questionnaire-based perception surveys are conducted in South Delhi to analyze socio-economic determinants of perceptional deviation. Memory span is studied in both controlled and field settings, and regression modeling is performed on the laboratory dataset.

2. Literature Review

2.1 Route Choice Modelling

Route choice modeling involves representing the choice of a route from a set of alternatives. As defined by Dial (1971), valid routes in the choice set should progress travelers from origin toward destination. Akiyama et al. (1992) demonstrated through laboratory experiments that travelers' choices are highly influenced by prior experiences and the amount of information available; groups with more information exhibited flow variations closer to equilibrium. Spiess and Florian (1989) established the optimal strategy concept, in which passengers minimize expected travel time by adopting strategies rather than fixed paths, under the assumption of infinite vehicle capacity.

De Cea and Fernandez (1993) advanced the field by developing an equilibrium model for congested public transport systems. They modeled congestion as additional waiting time at stops, expressed through a BPR-type cost function, and solved a numerical example using the Frank-Wolfe algorithm. Teklu (2008) extended this by applying the Monte Carlo-based SIMTRANSIT model in a day-to-day framework, comparing its outputs with the De Cea and Fernandez analytical model.

2.2 Stochastic User Equilibrium

Wardrop's two principles—User Equilibrium (UE) and System Optimum (SO)—form the theoretical basis for traffic assignment. The unrealistic assumption that all travelers have perfect network knowledge motivated the development of Stochastic User Equilibrium (SUE), introduced by Daganzo and Sheffi (1977), in which no traveler believes they can improve their travel time by unilaterally switching routes. The probability of choosing a route depends on the distribution of random components representing perceptual variation.

Sheffi and Powell (1982) compared stochastic (probit-based) and deterministic assignment over congested networks, finding that UE and SUE flows become similar as congestion increases. Prashker and Bekhor (2000) confirmed that SUE approaches SO for lightly congested networks, while UE and SUE converge under heavy congestion. Chen and Alfa (1991) proposed algorithms with optimized step-length determination to improve convergence over the Method of Successive Averages (MSA). Maher (1998) developed the Optimal Step Length Algorithm (OSLA) for logit-based SUE, while Liu et al. (2009) proposed the Method of Successive Weighted Averages (MSWA) and self-regulated averaging methods as practical alternatives.

2.3 Day-to-Day Models

Day-to-day (DTD) models capture the evolution of network flows over successive days as travelers learn and adapt. Hazelton and Parry (2015) highlighted their advantages in network management and behavioral analysis. Horowitz (1984) formalized three learning process models in which passengers update perceptions based on historical experiences with non-negative weights. Avineri and Prashker (2006) found through laboratory experiments that learning is slower without static information and that travelers can be classified into risk-aversion categories.

Cascetta and Cantarella (1991) developed a stochastic day-to-day model incorporating within-day variability, using Monte Carlo simulation or fractional probabilities for route assignment. Maio et al. (2013) further extended DTD models by allowing dynamic choice set evolution, incorporating both directly experienced travel times and indirectly acquired network information. Iryo (2016) developed a macroscopic model from microscopic DTD dynamics, accounting for both personal experience and information from peers. Seetharaman (2017) proposed the Reliability-based Disaggregate Stochastic Process Model (R-DSPM), which used a weighted average of past experiences within a fixed memory length (15 days) for the logit model, though the value was not calibrated from data.

3. Methodology

The research methodology proceeds through the following sequential stages:

- Comprehensive review of literature on route choice modeling, SUE, and day-to-day models.
- Coding of a baseline day-to-day model for the dummy transit network in MATLAB.
- Coding of the SUE model using the MSA algorithm for the same network.
- Comparison of SUE converged flows with average flows from the DTD model.
- Identification of study routes and field data collection in South Delhi.
- Field perception survey (unfamiliar passengers) and questionnaire survey at bus stops (familiar passengers).
- Laboratory memory span tests using the forward digit span method.
- Regression analysis of perception deviation and memory span data in SPSS.

Both models are implemented in MATLAB R2014a. Perception survey data and memory test results are analyzed in Excel 2024 (v16.0) and SPSS. The within-day simulation interval is fixed at one hour, with a step length of 360 seconds and a 30-day burn-in period before analysis of DTD results.

4. Model Development

4.1 Network Description

The example network proposed by De Cea and Fernandez (1993) serves as the test bed for both models. The network consists of four nodes (origin: Node 1; destination: Node 4; transfer nodes: 2 and 3) and six route sections (S1–S6). Four bus lines operate on the network, each with a vehicle capacity of 10 passengers:

- Bus Line 1: Origin (Node 1) to Destination (Node 4), direct; frequency = 10 buses/hr; in-vehicle travel time = 25 min.
- Bus Line 2: Node 1 → Node 2 → Node 3; frequency = 10 buses/hr; in-vehicle time = 7 min (Nodes 1–2) + 6 min (Nodes 2–3).
- Bus Line 3: Node 2 → Node 3 → Node 4; frequency = 4 buses/hr; in-vehicle time = 4 min per section.
- Bus Line 4: Node 3 → Node 4; frequency = 20 buses/hr; in-vehicle time = 10 min.

Four routes are considered between Nodes 1 and 4:

Table 1: Routes and Corresponding Route Sections

Route	Path	Route Sections
Route 1	$1 \rightarrow 4$	S3
Route 2	$1 \rightarrow 3 \rightarrow 4$	S2, S6
Route 3	$1 \rightarrow 2 \rightarrow 4$	S1, S5
Route 4	$1 \rightarrow 2 \rightarrow 3 \rightarrow 4$	S1, S4, S6

4.2 Cost Function

Bus inter-arrival times are assumed to follow an exponential distribution; therefore, the average waiting time at a node is the reciprocal of the effective frequency of lines serving that section. For route sections served by multiple bus lines, the expected travel time is computed as a frequency-weighted average of in-vehicle times. The total cost for a route section incorporates free-flow travel time and additional congestion waiting time via the BPR function:

$$Cs = ts + \alpha/fs + \beta \cdot \varphi s((Vs + \bar{V}s)/Ks)$$

where ts is the expected in-vehicle travel time for section s, fs is the effective frequency, Vs is the boarding volume, $\bar{V}s$ is the competing volume sharing the same capacity, Ks is the section capacity, and α, β are calibration constants (set to 1 and 20 respectively, following De Cea and Fernandez (1993)).

The Jacobian matrix of the cost vector (first derivative of additional waiting time with respect to volume) is used to compute congestion costs efficiently in MATLAB.

4.3 Stochastic User Equilibrium (SUE) Model

The SUE model is solved using the Method of Successive Averages (MSA). Route choice probabilities follow the multinomial logit model:

$$Pr = \exp(-\theta Cr) / \Sigma j \exp(-\theta Cj)$$

where, Cr is the perceived cost of route r and θ is the logit dispersion parameter. The flow in route r is then $Vr = D \cdot Pr$, where D is the total O-D demand. The algorithm iterates through direction-finding (computing auxiliary flows from perceived travel times) and movement (updating flows with step size $\alpha = 1/p$ at iteration p) until convergence, tested using the criterion proposed by Chen and Alfa (1991) with $I = 5$ and $\varepsilon = 0.001$.

4.4 Day-to-Day (DTD) Model

In the DTD model, each of the D passengers generates a stochastic arrival time at the origin stop during the simulated one-hour peak period. Bus arrivals on each line are drawn from exponential distributions with means equal to the inter-arrival headways. At each 360-second step, each passenger's route choice is determined using the logit model applied to their predicted travel time for that day.

Travel time prediction employs a finite memory span. For the first p days (where p is the memory span), each traveler uses only the immediately prior day's experienced travel time. From day p + 1 onward, the predicted travel time is the average of the p most recent experiences:

$$T_r^n = (E_r^{n-1} + E_r^{n-2} + \cdots + E_r^{n-p}) / p$$

where T_r^n is the predicted travel time for route r on day n, p is the memory span, and E_r^g is the experienced travel time on day g. A random component drawn from a standard normal distribution is added to the congested cost to convert actual travel time to perceived travel time, introducing stochasticity at the individual level. En-route switching is not permitted; once a route is chosen, the traveler completes that route. The model is run for 150 days; the first 30 days are treated as a burn-in period and excluded from analysis. Sensitivity tests are performed with memory spans $p \in \{4, 7, 10\}$ and demands $D \in \{100, 200, 240\}$.

5. Data Collection

5.1 Study Area

The study area is bounded by four bus stops in South Delhi: Ashram Chowk, Badarpur Border, Mehrauli, and AIIMS. Services operated by the Delhi Transport Corporation (DTC) within this corridor were selected. Five O-D pairs with two competing bus lines each were identified through a pilot survey conducted on January 19, 23, 24 and February 1, 8, 2026. Routes with travel times below 10 minutes or above one hour were excluded.

Table 2: O-D Pairs with Selected Bus Lines and Travel Times

Origin–Destination	Distance (km)	Bus Lines	Travel Time (min)
Nehru Nagar – Govindpuri Metro	4.0	429 / 469	16 ± 3.5 / 14.3 ± 1.2
Tara Apartment – LSR College	5.1	425 / 445A	37.7 ± 6.8 / 35.7 ± 0.8
Suraj Kund Crossing – Govindpuri	6.3	433 / 511A	21.0 ± 2.0 / 23.7 ± 5.7
NSIC – Masjid Moth	3.4	427 / 534A	11.5 ± 3.6 / 12.0 ± 5.3
Ashram – Aali Village	10.2	405 / 479	37.7 ± 6.8 / 35.7 ± 0.8

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5.2 Perception Survey — Field Survey

A field perception survey was conducted on February 19–21 and 27–28, 2026. Fifty-one participants who were unfamiliar with the selected routes made a minimum of ten trips each on the designated O-D pairs during morning (8:00 am–12:00 pm) and evening (4:00 pm–8:00 pm) peak hours. Surveyors stationed at the origin stop issued tokens, while those at the destination recorded perceived travel times reported by passengers. Participants were not permitted to note actual boarding or alighting times, ensuring that subjective perception rather than objective measurement was captured. Perceived travel times for random trips were later recalled to check memory accuracy.

5.3 Questionnaire Survey at Bus Stops

A separate questionnaire-based survey was conducted at three bus stops (Suraj Kund Crossing, Govindpuri Metro Station, and Ashram Bus Stop) on February 19–21, 2026. Approximately 50 familiar passengers per stop were interviewed. Familiarity was verified by asking respondents to correctly identify the bus numbers serving the route. Average perceived travel time for the route was recorded, along with socio-economic information.

5.4 Memory Span Test

Memory span was assessed using the forward digit span test, the most commonly used measure of short-term memory (Martin, 1978). Sets of three random digits were read to participants at a rate of one per second, and they were asked to recall the digits in order. The sequence length was progressively increased from 2 to 10 digits. The span was recorded as the longest sequence correctly recalled; partial credit was awarded (e.g., if a participant correctly recalled 2 out of 3 items at span 5, their span was scored as 4 + 2/3, rounded to the nearest integer).

Tests were conducted during off-peak hours in the CRRI Environmental Division laboratory—a controlled, low-distraction environment—for the 47 participants of the field survey. Memory was also assessed in the field by asking participants to recall perceived travel times and corresponding trip numbers, with a tolerance of ±5–10 minutes.

6. Data Analysis and Results

6.1 Perception Survey Analysis — Field Survey

Twenty-five participants completed the minimum of ten trials. The sample was approximately gender-balanced (52% male, 48% female); 52% were aged 18–30 and 44% aged 31–50; 80% had an annual income below ₹1 lakh; and 52% held matriculate-level education. About 48% were daily bus users and 40% traveled 2–3 times per week.

Multiple linear regression with perceptional deviation from the mean route travel time as the dependent variable yielded an R^2 of 0.694, indicating that approximately 69% of the variance in deviation is explained by the independent variables. Key findings:

- Men show less deviation than women in perceiving travel time for unfamiliar routes.
- Perceptional deviation increases with age.
- Higher educational qualification reduces deviation.

- Frequency of bus use has a strong positive influence: regular travelers exhibit significantly lower deviation than occasional users.

Table 3: Regression Results — Perceptional Deviation (Field Survey)

Variable	Description	Coefficient
GE	Gender (1=Male, 2=Female)	1.5391
AG	Age category	2.1289
IN	Annual income category	0.1480
ED	Educational qualification	−0.6703
US	Frequency of bus use	1.0869
R^2	—	0.6938
Adj. R^2	—	0.6853

The regression equation for perceptional deviation is:

$$\text{Deviation} = 1.5391(GE) + 2.1289(AG) + 0.1480(IN) - 0.6703(ED) + 1.0869(US)$$

6.2 Perception Survey Analysis — Questionnaire Survey

Forty-three reliable samples were collected from familiar passengers. The sample was gender-balanced (51% male, 49% female); 53% were aged 31–50 and 42% aged 18–30; 51% earned ₹1–5 lakh annually; 44% were illiterate and 30% were graduates. About 60% were daily commuters. Regression analysis ($R^2 = 0.818$) for familiar passengers revealed the following:

- Perceptional deviation increases with age and is higher for women in the sample.
- Higher income and education reduce deviation—unlike the field survey, income shows a negative coefficient here, reflecting that higher-income familiar passengers have a greater value of time and are more attentive to travel time.
- Frequency of bus use on the specific route has the highest influence on deviation.

Table 4: Regression Results — Perceptional Deviation (Questionnaire Survey)

Variable	Coefficient
GE (Gender)	0.0391
AG (Age)	0.2511
IN (Income)	−0.3448

ED (Education)	−0.8021
US (Frequency of use)	5.0658
R^2	0.8176

6.3 Memory Span Analysis

Data from the digit span test conducted on 47 participants (62% male, 38% female; 66% aged 18–30) were analyzed. Memory span was taken as the dependent variable. Correlation analysis showed the highest positive correlation between gender and education (0.49), with negative correlations between age and both education (−0.60) and gender (−0.62).

Regression analysis yielded an R^2 of 0.913 (Adj. R^2 = 0.884), indicating an excellent model fit. Key findings:

- Memory span decreases with age, consistent with established neuroscience literature.
- Men exhibit higher digit spans than women in the sample.
- Both income and educational qualification are positively associated with memory span; education exerts a stronger effect than income.

Table 5: Regression Results — Memory Span (Laboratory)

Variable	Coefficient
GE (Gender: 1=Female, 2=Male)	0.4318
AG (Age category)	−0.0263
IN (Income category)	0.3946
ED (Education level)	1.0771
R^2	0.9130
Adj. R^2	0.8836

The regression equation for memory span is:

$$\text{Memory Span} = 0.4318(GE) - 0.0263(AG) + 0.3946(IN) + 1.0771(ED)$$

6.4 Laboratory vs. Field Memory Span Comparison

A systematic comparison of memory span values obtained from the controlled laboratory test and the field assessment reveals that the field memory span is consistently lower than the laboratory span for virtually all 47 individuals tested. The average laboratory digit span was approximately 4, while the average field span was approximately 2. Despite this gap, a positive correlation exists between the two values: individuals with higher laboratory spans also tended to recall more travel times accurately in the field.

This discrepancy is attributed to extrinsic distraction factors present in real-world transit settings (noise, crowding, and cognitive load of actual travel) that reduce the effective utilization of an individual's full memory capacity. The finding provides empirical justification for using a conservatively calibrated memory span in the DTD model, rather than the raw laboratory value.

6.5 SUE Model Results and Sensitivity Analysis

The SUE model was run for demand levels $D \in \{100, 200, 240\}$ and dispersion parameters $\theta \in \{0.01, 0.05, 0.1, 0.5\}$. Across all cases, Route 1 (direct, 1→4) consistently attracted the highest flow, and Route 4 (longest, 1→2→3→4) received the lowest flow. Selected results for D = 240 are summarized below:

Table 6: SUE Flow Distribution for Demand = 240 (various θ)

θ	Route 1 Flow	Route 2 Flow	Route 3 Flow	Route 4 Flow
0.01	152	58	17	13
0.05	137	84	16	3
0.10	141	80	18	1
0.50	134	92	14	0

As θ increases, travelers become increasingly sensitive to travel time differences, concentrating flow on the shortest routes (Routes 1 and 2) and reducing flow on longer routes. At $\theta = 0.5$, Route 4 receives zero flow for all demand levels, while Route 3 retains a small non-zero flow due to its intermediate travel time. This pattern confirms the theoretical result that the logit model approaches deterministic assignment as $\theta \to \infty$. All route flows converge to stable SUE values, with convergence achieved faster for higher θ values.

6.6 Day-to-Day Model Results

The DTD model was simulated for 150 days across all combinations of $D \in \{100, 200, 240\}$ and $p \in \{4, 7, 10\}$. Unlike the SUE model, individual daily flows fluctuate stochastically and do not converge to a fixed equilibrium—a realistic reflection of real network behavior. Route 1 consistently has the highest average flow, and Route 4 the lowest, mirroring the SUE hierarchy. Selected results for D = 240 (memory span p = 7, days 30–150) are summarized below:

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Table 7: DTD Flow Distribution for D = 240, Memory Span = 7

Day	Route 1	Route 2	Route 3	Route 4
30	134	41	40	25
50	119	50	44	27
75	161	52	18	9
100	85	74	36	45
125	129	36	51	24
150	160	30	20	30
Average (Days 31–150)	138	49	29	24

A critical finding is that the average flow in each route over the post-burn-in period (days 31–150) closely matches the converged SUE flow for the same demand and comparable θ. This validates the DTD framework as a dynamic counterpart to SUE: while equilibrium is never exactly realized on any single day, the system's time-averaged behavior is consistent with the SUE solution.

The effect of memory span p is also notable: as p increases from 4 to 10, route flows become more stable over time, with reduced day-to-day variance. Passengers with longer memory spans accumulate more reliable average travel time estimates and are therefore less likely to switch routes based on single aberrant experiences. This finding supports the utility of calibrating the memory span parameter to empirical data.

7. Discussion

This study makes several conceptual and empirical contributions to the transit assignment literature. First, the explicit incorporation of a finite memory span parameter into a microscopic DTD framework bridges an important gap: previous DTD models either used infinite memory (full history) or an uncalibrated constant. The empirical finding that field-effective memory span (~2) is substantially lower than laboratory span (~4) highlights the importance of calibrating this parameter from real-world data rather than assuming theoretical values.

Second, the perception survey results reveal that socio-economic variables—particularly education, frequency of transit use, and gender—significantly influence the accuracy of travel time perceptions. These findings can inform the design of more heterogeneous agent populations in future DTD models, where different traveler segments are assigned different dispersion parameters and memory spans based on their demographic profiles.

Third, the comparison between DTD and SUE models demonstrates that while equilibrium-based models are computationally tractable and provide valid representations of average system behavior, they cannot capture the day-to-day flow variability inherent in real networks. The DTD model's stochastic flow evolution better represents reality and enables analysis of transient phenomena such as the impact of service disruptions, information campaigns, or policy interventions.

The finding that higher θ (greater sensitivity to travel time) concentrates flow on Route 1 and progressively empties Route 4 is consistent with theoretical predictions and with the intuition that more rational travelers will gravitate toward the objectively faster route. The persistence of flow on Route 3 even at high θ reflects its competitive travel time relative to Route 4.

8. Conclusions

This paper developed and validated a microscopic day-to-day transit assignment model incorporating the memory length parameter, and benchmarked it against a conventional SUE model on the De Cea and Fernandez (1993) dummy network. The following conclusions are drawn:

- Men exhibit lower perceptional deviation from mean travel time than women for unfamiliar routes; deviation increases with age and decreases with education and frequency of transit use.
- For familiar route passengers, higher income reduces perceptional deviation—reflecting a greater value of time and heightened attention to travel time accuracy.
- Memory span in controlled laboratory conditions (average ~4) significantly exceeds the span effectively utilized in real-world field conditions (average ~2), due to environmental distractions. The positive correlation between laboratory and field spans suggests that laboratory tests can serve as an ordinal predictor of field capacity.
- Memory span is positively associated with educational qualification and income, and negatively with age. Educational qualification exerts the strongest influence among the explanatory variables considered.
- In the DTD model, individual daily flows are stochastic and do not converge to a fixed value, reflecting real network behavior. However, the time-averaged flows across days 31–150 are comparable to the converged SUE flows for the same demand.
- As memory span increases ($p = 4 \rightarrow 10$), route flows stabilize, exhibiting reduced day-to-day variance. This supports the behavioral interpretation that travelers with longer memory spans have more reliable travel time forecasts.
- In both models, Route 1 (direct) consistently attracts the highest flow and Route 4 (longest path) the lowest. As θ increases, flow becomes increasingly deterministic, with Route 4 flow reaching zero at $\theta = 0.5$ for all demand levels.

8.1 Limitations

- The logit dispersion parameter θ is assumed constant across all travelers; in reality it varies with individual risk attitudes and familiarity.
- Memory span is assumed homogeneous across all travelers in the DTD model; heterogeneous memory spans would more realistically represent the population.
- En-route switching is not modeled; passengers are locked into their chosen route once a trip begins.
- The study network is a small dummy network; validation on real-world multi-route networks remains to be conducted.

8.2 Future Scope

- Apply the developed DTD model to real transit networks with heterogeneous memory spans calibrated from empirical data.
- Incorporate the impact of static or dynamic traveler information on perception updating.
- Extend the model to include departure time choice in addition to route choice.
- Use more sophisticated neuropsychological methods to measure the effective memory span utilized in transit contexts.
- Develop heterogeneous agent models in which individual memory spans and dispersion parameters are drawn from empirically calibrated distributions.

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